

# Electrical Measurements

Code: EPM1202

Lecture: 4

Tutorial: 2

Total: 6

Dr. Ahmed Mohamed Azmy

Department of Electrical Power and Machine Engineering  
Tanta University - Egypt



Faculty of  
Engineering



Tanta University

# Indicating measuring instruments

## The moving coil instruments

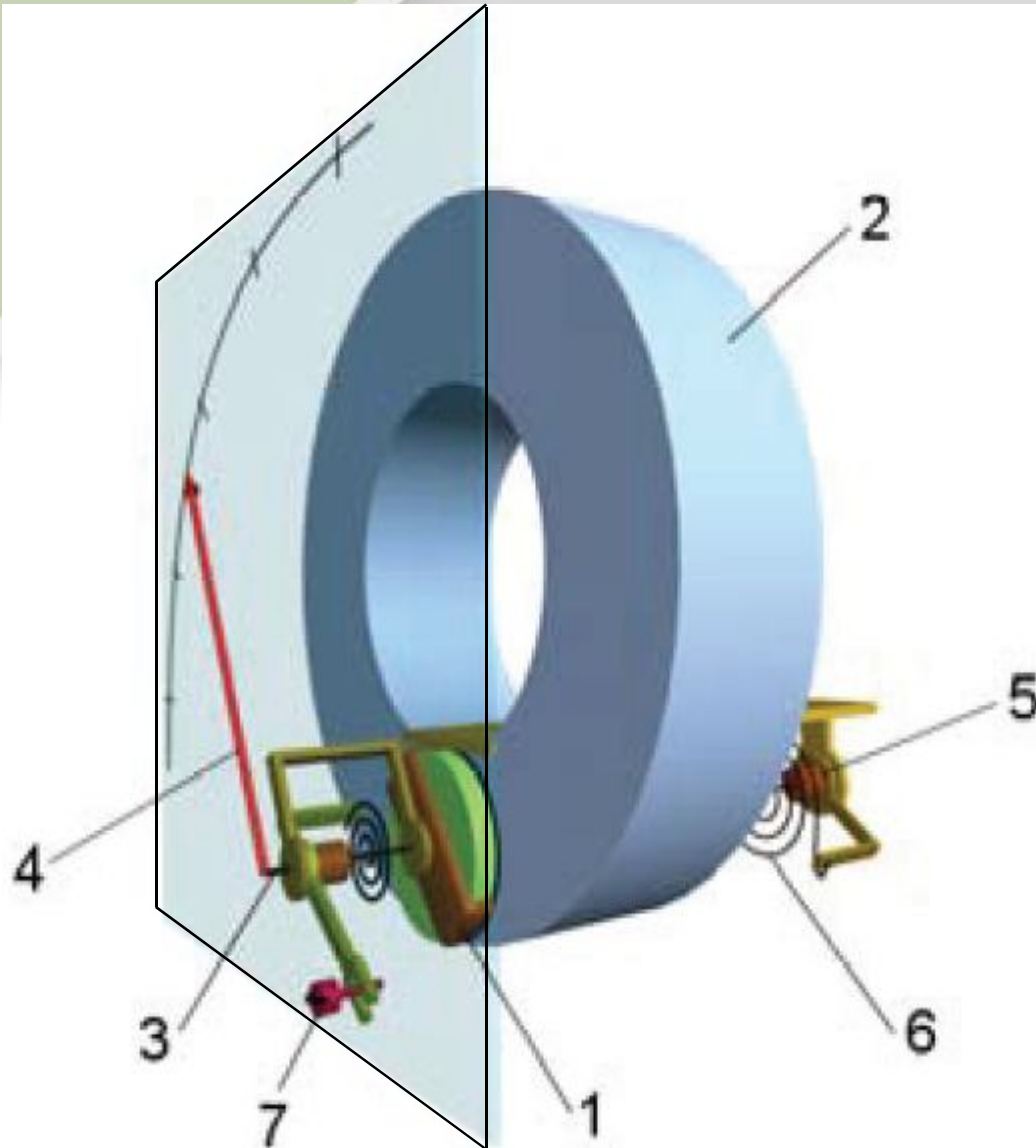
The Permanent-Magnet Moving-Coil (PMMC) instrument “d'Arsonval meter movement” is the most-used indicating electro-mechanical device

It is the most accurate type for direct current measurements “only for direct current measurements”

The measured value is obtained using a pointer, which indicates the reading on a calibrated scale

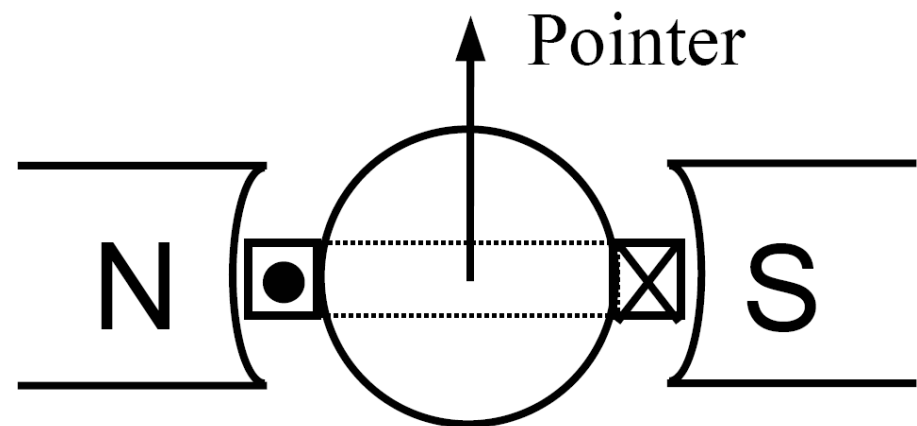
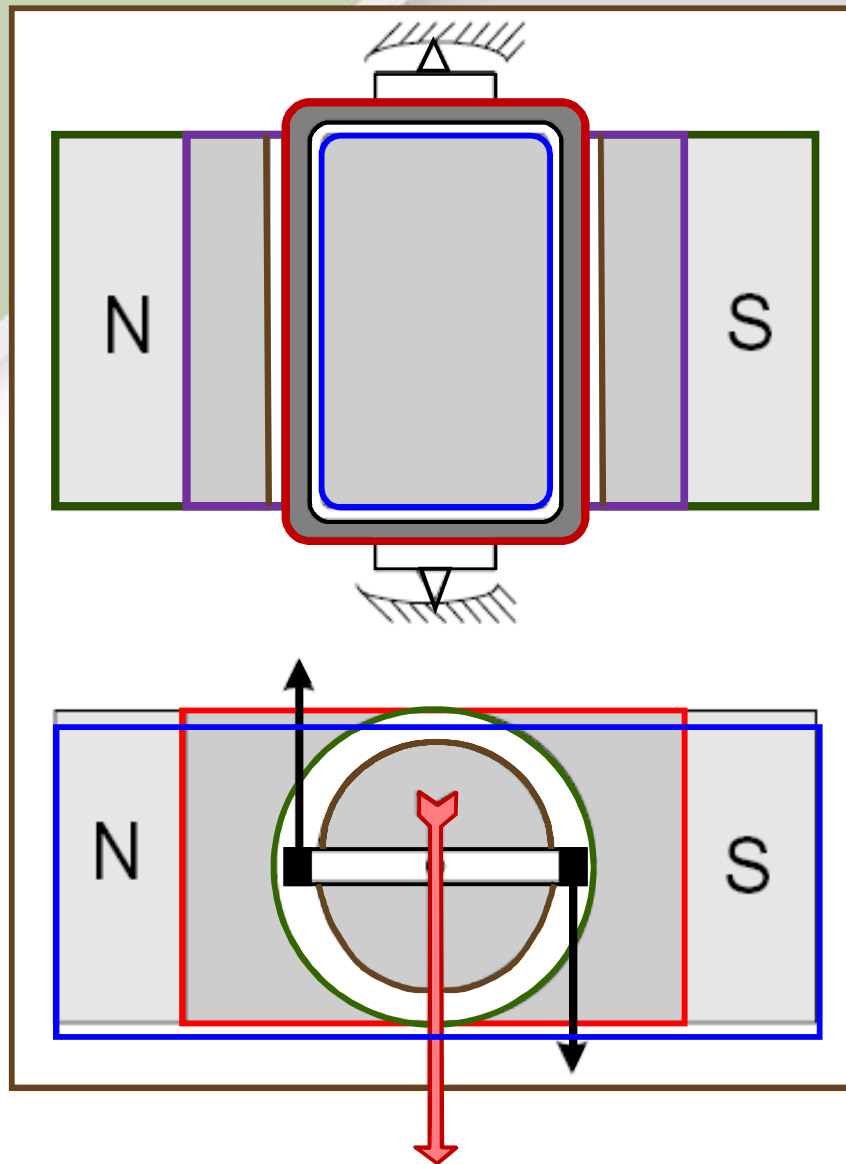
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# The moving coil instruments

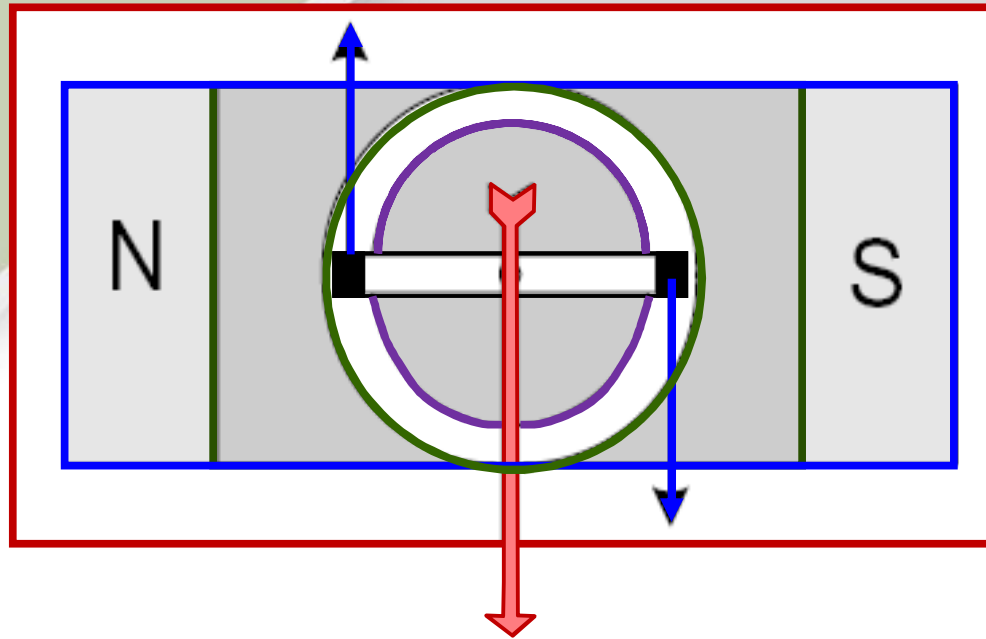


- 1- moving coil
- 2 – permanent magnet
- 3 – axle
- 4 – pointer
- 5 – bearings
- 6 – spring
- 7 – correction of zero

# The moving coil instruments



## The moving coil instruments



The pointer is fixed to the axle of a rectangular coil

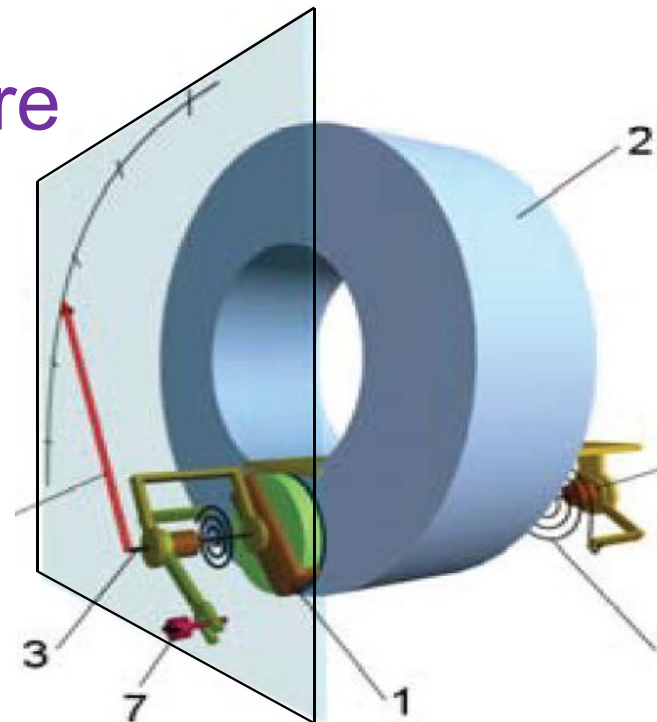
This coil is used as the moving part in the instruments

The current is delivered to the coil and two springs are used as the mechanisms generating control torque

# The moving coil instruments

## Components of moving coil instruments

- A permanent magnet, shaped as horseshoe
- Poles manufactured from soft iron
- Cylindrical core from soft iron
- Moving coil made from fine wire wound on a light metal frame
- Bearings
- A pointer
- A scale
- A spiral spring



# The moving coil instruments

## Operation principles

The moving coil is placed in a gap between the magnet poles

The coil movement is due to the interaction between the magnetic field of the magnet and the magnetic field generated by the coil

The magnetic field of the coil is caused due to the flowing of the current through the coil

The rotation of the coil, and hence the pointer, is due to the torque “T”

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# The moving coil instruments

## Operation principles

It is important for dc meter to indicate the polarity of the instrument on its terminals

Moving-coil instrument responds to the current rather than any other variable

When reading other quantities, e.g. voltage, it is required to calibrate the scale to the proper unit to be measured

When a current of “ $I$ ” flows through a coil with “ $N$ ” turns, a force “ $F$ ” is produced on each coil side



# The moving coil instruments

## Operation principles

The produced force is given as:

$$F = B \cdot N \cdot \ell \cdot I \quad \text{Newton}$$

The deflection torque depends on the flux density “B”, on dimensions “d” and “ℓ” of the coil, on the number of turns “N” and on the measured current “I”:

$$T = d \cdot \ell \cdot B \cdot N \cdot I \quad \text{N.m}$$

$$T = K \cdot I, \quad K = d \cdot \ell \cdot B \cdot N = a \cdot B \cdot N$$

# The moving coil instruments

## Operation principles

The position of the moving element results from the balance between the torque and the controlling torque of the springs

$$T = T_c$$

The controlling torque is in proportional with the deflecting angle “ $\theta$ ”

$$C\theta = d.\ell.B.N.I \quad \Rightarrow \quad \theta = \frac{d.\ell.B.N}{C} I$$

$$\theta \propto I$$

# The moving coil instruments

## Operation principles

$$\theta \propto I$$

The moving coil has a linear scale and the divisions are equally spaced

With large proportional constant, the instrument will have high sensitivity since less current will produce more movement of the coil

Large proportional constant is obtained by large magnetic flux density “B”

The increase of the number of turns or the dimensions of the coil will result in an increase in the weight and the resistance of the coils

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# The moving coil instruments

## Operation principles

PMMC instruments consume very low power in the range between  $20 \mu\text{W}$  and  $250 \mu\text{W}$

The accuracy of such instruments lies between 2% to 5% at full scale

PMMC instruments are sensitive to the temperature variation

Special arrangements are required to compensate for the effect of temperature

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# The moving coil instruments

## Operation principles

PMMC instrument can measure ac signals but the current has to be rectified

If ac signals are directly applied to PMMC instruments, the pointer can not follow up the fast variation of the power due to the high frequency

The pointer will read the average value of the signal, which is zero in this case, and hence it will oscillate around the zero value

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## Advantages of moving coil instruments

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graph TD; A[Advantages of moving coil instruments] --> B[Low power consumption]; A --> C[High torque/weight ratio]; A --> D[Used for a wide ranges of currents and voltages]; A --> E[Uniform scales]; A --> F[Not strongly affected by the external stray magnetic fields]; B --> G[They have no hysteresis losses]; E --> H[Not strongly affected by the external stray magnetic fields];
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Low power  
consumption

High torque/  
weight ratio

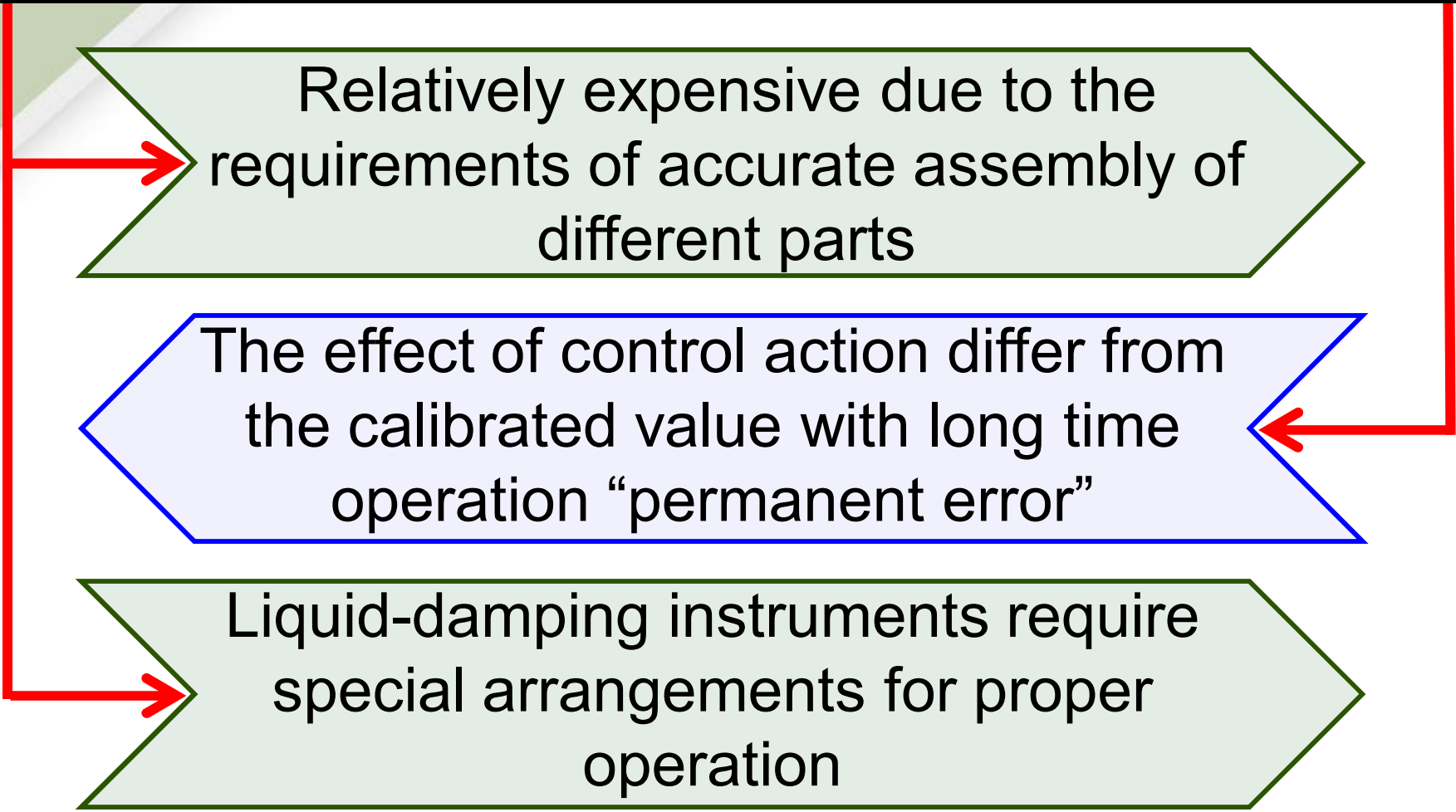
They have no hysteresis  
losses

Used for a wide  
ranges of currents  
and voltages

Not strongly affected  
by the external stray  
magnetic fields

Uniform  
scales

## Disadvantages of moving coil instruments



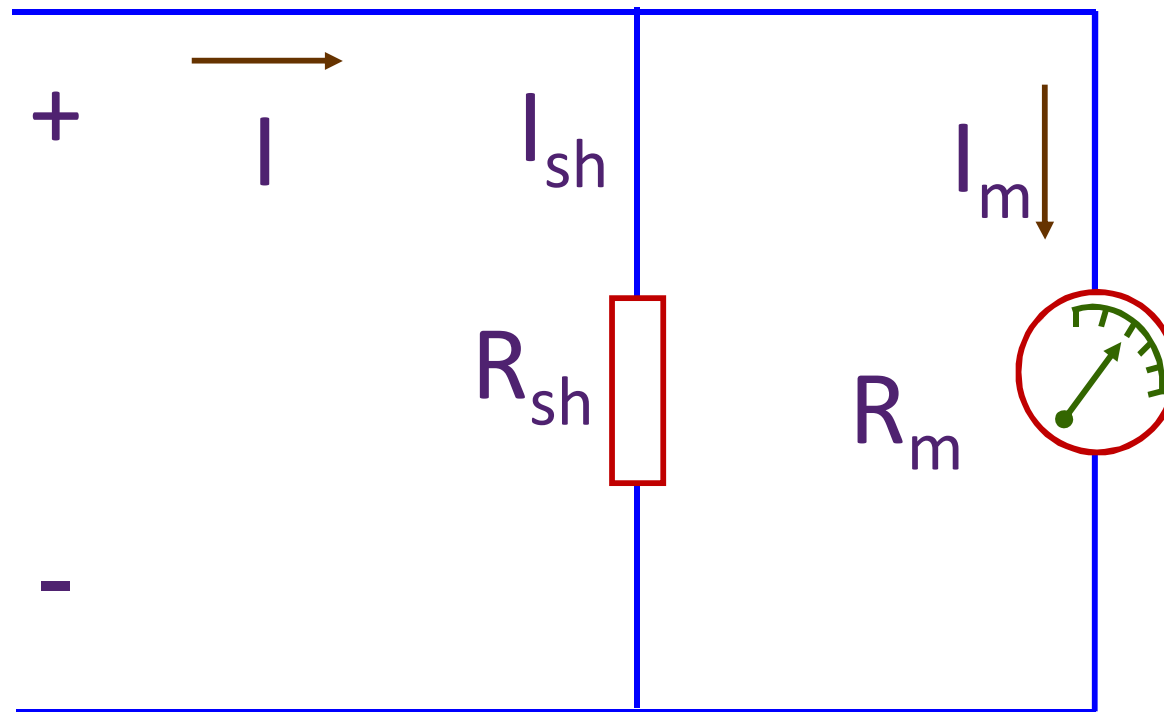
Relatively expensive due to the requirements of accurate assembly of different parts

The effect of control action differ from the calibrated value with long time operation “permanent error”

Liquid-damping instruments require special arrangements for proper operation

# The moving-coil instrument as an ammeter

The moving-coil instrument is connected in parallel with a shunt resistor





# The moving-coil instrument as an ammeter

The practical manufacture of the windings requires that the resistance is relatively high

It is necessary to connect the shunt resistance to reduce the overall resistance to a suitable value

The reading depends on the current in the meter  $I_m$

The measured current is the total current “I”

The instrument has to be calibrated to indicate the total current “I” instead of the meter current “ $I_m$ ”

The meter current “ $I_m$ ” is proportional to total current

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# The moving-coil instrument as an ammeter

Normally, the shunt resistance is very low compared to the internal meter resistance

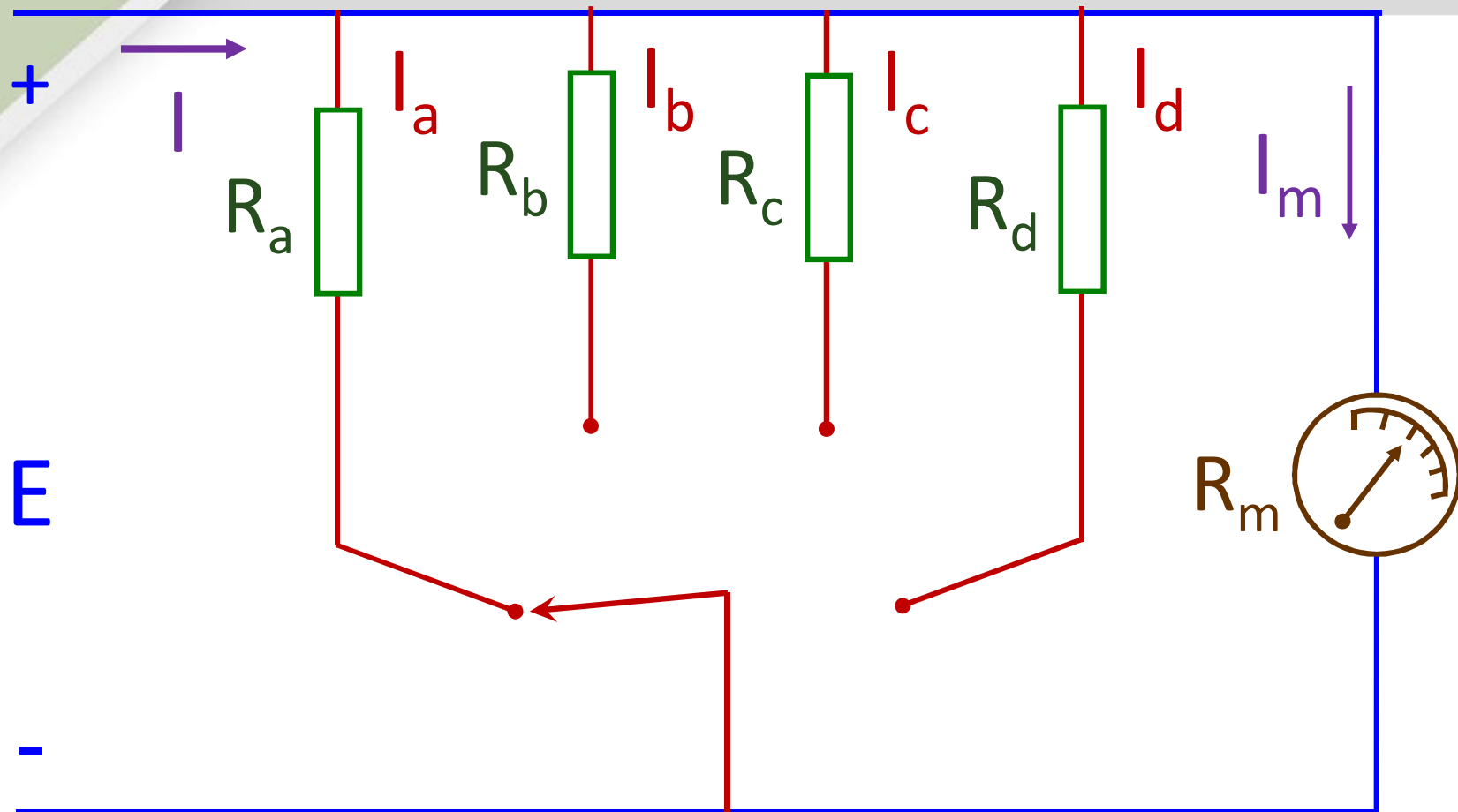
It is possible to measure high currents without any modifications (most current flows in shunt resistor)

The instrument can be used to measure high currents with very small current carrying capacity

It is possible to extend the operation of the instrument to have multi-range operation

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# The moving-coil instrument as an ammeter



# The moving-coil instrument as an ammeter

$$V_m = I_m \cdot R_m$$

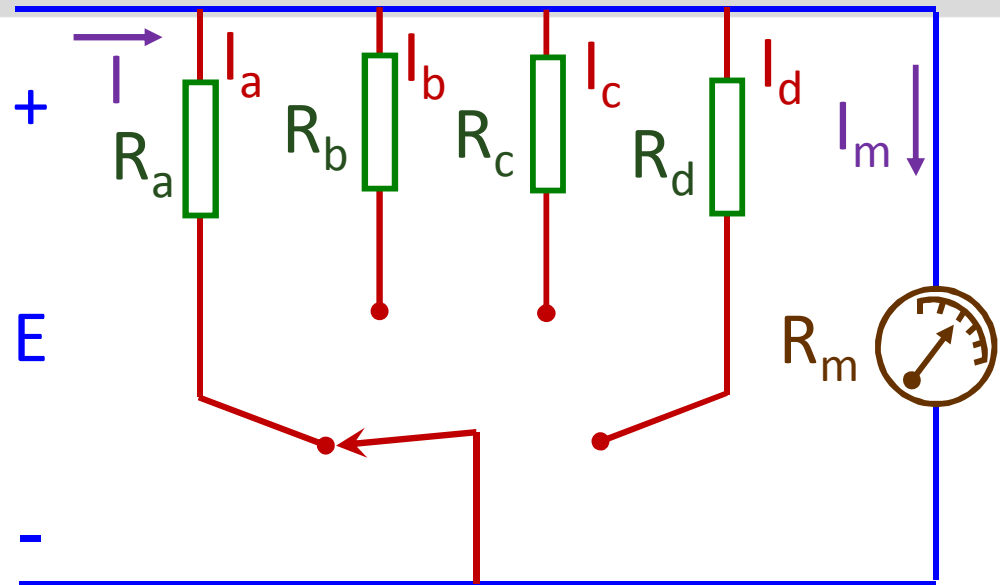
$$V_{sh} = I_{sh} \cdot R_{sh}$$

$$V_{sh} = V_m \rightarrow I_{sh} R_{sh} = I_m R_m$$

$$I_{sh} = I - I_m$$

$$I_{sh} \cdot R_{sh} = I_m \cdot R_m \rightarrow (I - I_m) \cdot R_{sh} = I_m \cdot R_m$$

$$R_{sh} = \frac{I_m}{I - I_m} R_m$$



# The moving-coil instrument as an ammeter

$$R_{sh} = \frac{I_m}{I - I_m} R_m = \frac{1}{\frac{I}{I_m} - 1} R_m = \frac{1}{n - 1} R_m$$

$$R_{sh} = \frac{1}{n - 1} R_m$$

“n” is a multiplying factor that correlates the total measured current and the meter full-scale current

# The moving-coil instrument as an ammeter

The temperature variations influence the flux density “B” of the permanent magnet and the elasticity of the springs

Fortunately, both of these influences result in opposite changes of the deflection angle “ $\theta$ ”

Therefore, their influences are negligible when the device is used as a microammeter

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# The moving-coil instrument as an ammeter

## **Example:**

A moving-coil instrument gives a full-scale deflection when the current is 40 mA and its resistance is 25  $\Omega$ . Calculate the value of the shunt to be connected in parallel with the meter to enable it to be used as an ammeter for measuring currents up to 50 A.

## **Solution:**

$$n = 50 / 0.04 = 1250$$

$$R_{sh} = \frac{1}{n-1} R_m = \frac{1}{1250-1} * 25 = 0.02002 \Omega$$

# The moving-coil instrument as an ammeter

## Example:

A 100  $\mu\text{A}$  moving-coil ammeter with an internal resistance of 800  $\Omega$  is used with a shunt resistor of 0.8 $\Omega$ . Find the maximum current that can be measured using this instrument

## Solution:

$$R_{sh} = \frac{1}{n-1} R_m \quad \Rightarrow \quad 0.8 = \frac{1}{n-1} * 800$$

$$n - 1 = 1000 \rightarrow n = 1001$$

$$n = I / 10^{-4} \rightarrow I = 0.1001 \text{ A}$$



# The moving coil instruments

## The Ayrton shunt

The Ayrton shunt is a special form of the shunt resistors used for multi-range ammeters

It can comprise as resistors as required to provides different ranges

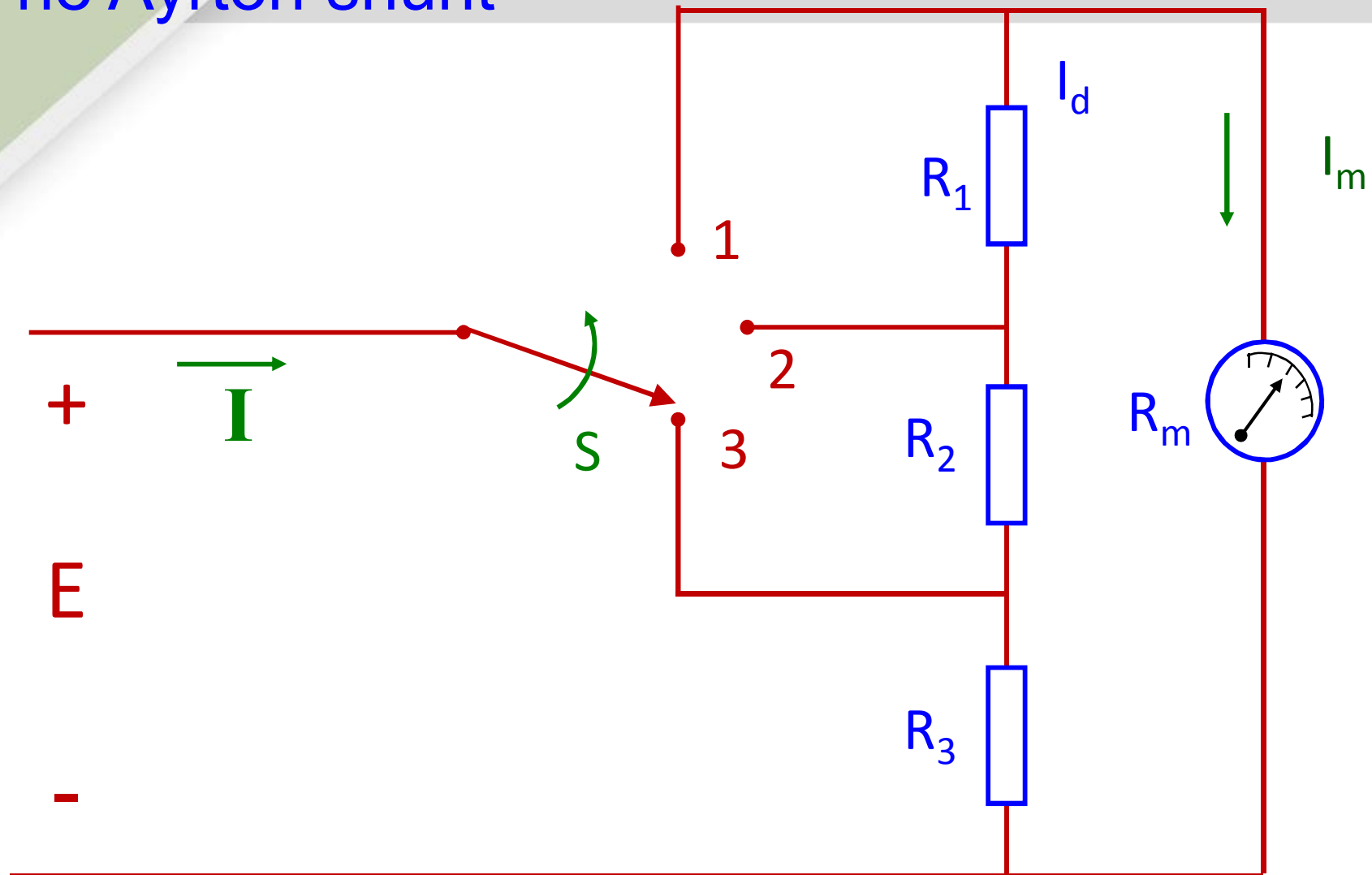
There is always a shunt resistor at any position of the switch

A wide range of measurements is possible with the appropriate design and the suitable number of shunt resistors

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# The moving coil instruments

## The Ayrton shunt



# The moving coil instruments

## The Ayrton shunt

### Position 1:

$$R_{sh} = R_1 + R_2 + R_3$$

$$n = I_1 / I_m$$

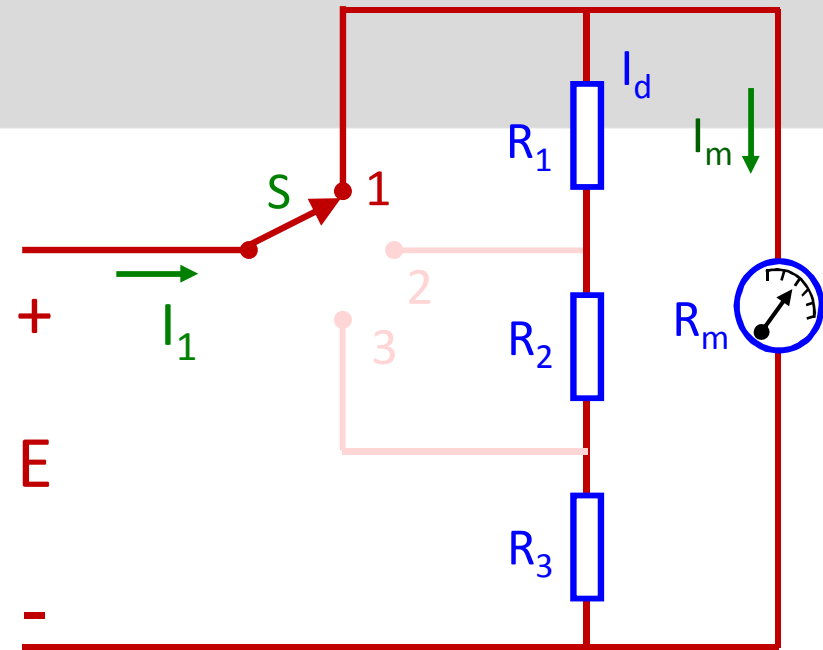
$$R_{sh} = R_m / (n-1)$$

Voltage across  $(R_1 + R_2 + R_3)$   
= Voltage across meter resistance

$$(R_1 + R_2 + R_3) * (I_1 - I_m) = I_m * R_m$$

$$R_1 + R_2 + R_3 = \frac{I_m}{I_1 - I_m} R_m$$

$$R_{sh} = \frac{1}{n-1} R_m$$



# The moving coil instruments

## The Ayrton shunt

### Position 2:

$$R_{sh-2} = R_2 + R_3$$

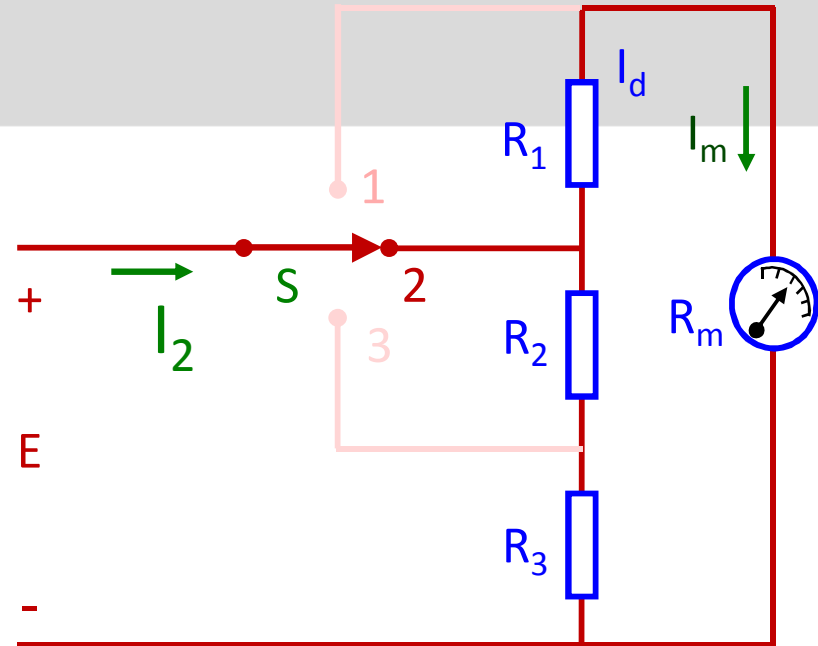
Voltage across  $(R_2 + R_3)$   
= Voltage across  $(R_m + R_1)$

$$(R_2 + R_3) * (I_2 - I_m) = I_m * (R_m + R_1)$$

$$(R_2 + R_3) * I_2 = I_m (R_1 + R_2 + R_3) + I_m * R_m$$

$$(R_2 + R_3) * I_2 = I_m R_{sh} + I_m * R_m$$

$$R_2 + R_3 = \frac{I_m}{I_2} (R_{sh} + R_m)$$



# The moving coil instruments

## The Ayrton shunt

### Position 3:

$$R_{sh-3} = R_3$$

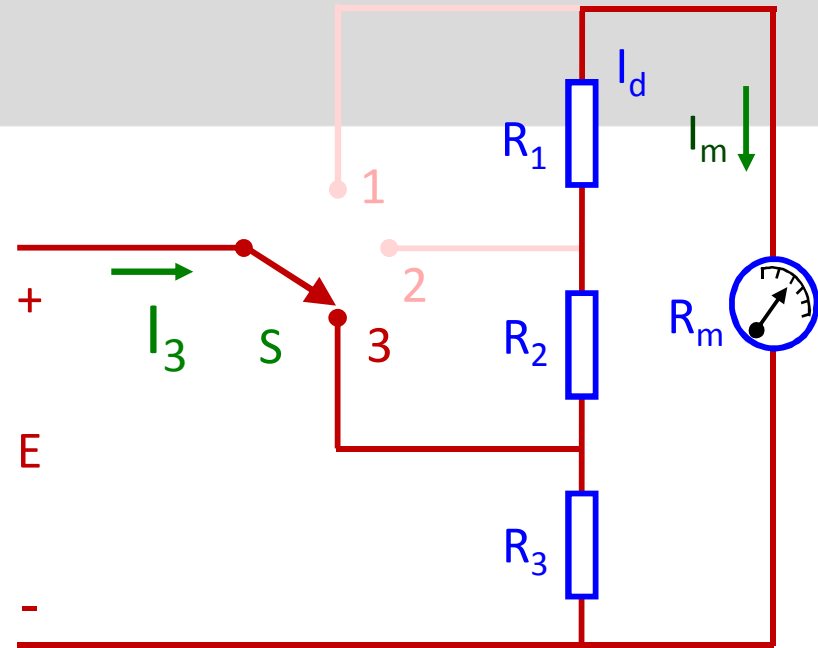
Voltage across ( $R_3$ ) =

Voltage across ( $R_m + R_1 + R_2$ )

$$R_3 * (I_3 - I_m) = I_m * (R_m + R_1 + R_2)$$

$$R_3 * I_3 = I_m (R_1 + R_2 + R_3) + I_m * R_m$$

$$R_3 * I_3 = I_m R_{sh} + I_m * R_m$$



$$R_3 = \frac{I_m}{I_3} (R_{sh} + R_m)$$

# The moving coil instruments

## The Ayrton shunt

- The multiplying factor “n” is calculated,  $n = I_1 / I_m$
- The total shunt resistance is derived,  $R_{sh} = R_m / (n-1)$
- The resistance “ $R_3$ ” is calculated

$$R_3 = \frac{I_m}{I_3} (R_{sh} + R_m)$$

- The resistance “ $R_2$ ” is then calculated

$$R_2 + R_3 = \frac{I_m}{I_2} (R_{sh} + R_m)$$

- The resistance “ $R_1$ ” is calculated

$$R_1 + R_2 + R_3 = \frac{I_m}{I_1 - I_m} R_m$$

**Example:** Design a three-section Ayrton shunt to be used with a meter that has an internal resistance of  $100\Omega$  and a full-scale deflection of  $1\text{ mA}$ . The meter should have three ranges of  $50\text{ mA}$ ,  $500\text{ mA}$  and  $5\text{ A}$

**Solution:**  $n = I_1 / I_m = 50 * 10^{-3} / 1 * 10^{-3} = 50$

$$R_{sh} = R_m / (n-1) = 100 / 49 = 2.04082 \Omega$$

$$R_3 = \frac{I_m}{I_3} (R_{sh} + R_m) = \frac{10^{-3}}{5} (2.04082 + 100) = 0.0204082 \Omega$$

$$R_2 + R_3 = \frac{I_m}{I_2} (R_{sh} + R_m) = \frac{10^{-3}}{0.5} (2.04082 + 100) = 0.204082 \Omega$$

$$R_2 = 0.204082 - 0.0204082 = 0.1836738 \Omega$$

$$R_1 = R_{sh} - (R_2 + R_3) = 2.04082 - 0.204082 = 1.836738$$

# The moving coil instruments

## Effect of ammeter insertion

The use of any instrument causes a certain error in the measurements depending on the connection method and the internal resistance

The additional resistance caused by the instrument still causes an error

The effect of inserting an ammeter is to increase the overall resistance of the circuit

The flowing current will be reduced

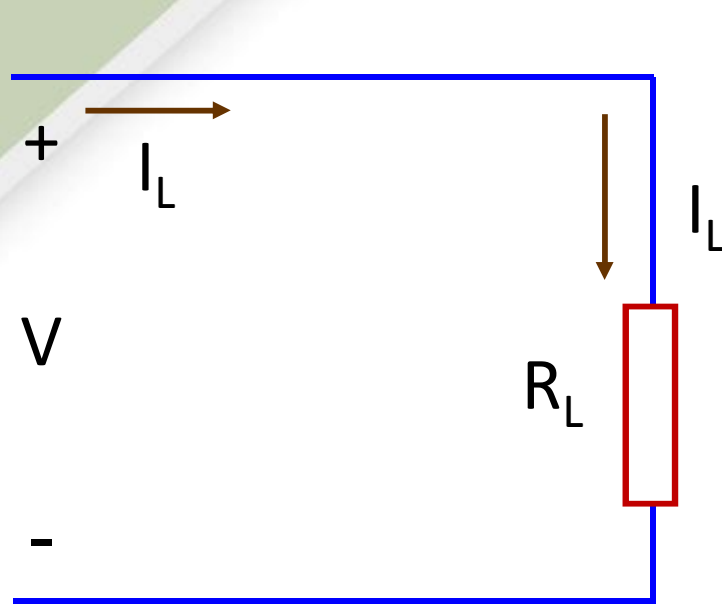
The instrument will read a current that is lower than the actual value

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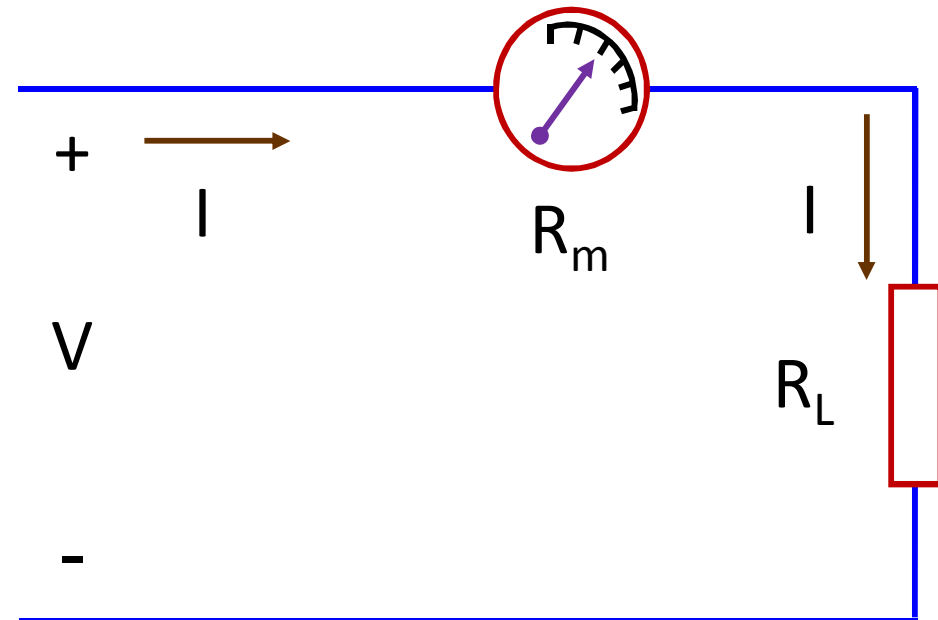
# The moving coil instruments

## Effect of ammeter insertion



Without ammeter

$$I_L = \frac{V}{R_L}$$



With ammeter

$$I = \frac{V}{R_L + R_m}$$

# The moving coil instruments

## Effect of ammeter insertion

$$error = \frac{V}{R_L} - \frac{V}{R_L + R_m}$$

$$\frac{I_m}{I} = \frac{R_L}{R_L + R_m}$$

**Example:** An ammeter has a full-scale deflection of 100mA and a resistance of  $50\Omega$ . The ammeter is used to measure the current in a load of  $500\Omega$  when the supply voltage is 10 V. Calculate (a) the ammeter reading expected (neglecting its resistance), (b) the actual current in the circuit, (c) the power dissipated in the ammeter, and (d) the power dissipated in the load

# The moving coil instruments

## Effect of ammeter insertion

### Solution:

(a) expected ammeter reading:

$$I = V / R = 10 / 500 = 0.02 \text{ A} = 20 \text{ mA}$$

(b) Actual ammeter reading:

$$I_m = V / (R + R_m) = 10 / (500 + 50) = 18.18 \text{ mA}$$

(c) Power dissipated in the ammeter

$$P_{\text{dis}} = I^2 * r_a = (18.18 * 10^{-3})^2 * 50 = 16.53 \text{ mW}$$

(d) Power dissipated in the load resistor

$$P_L = I^2 * R = (18.18 * 10^{-3})^2 * 500 = 165.3 \text{ mW}$$

# The moving-coil instrument as a voltmeter

The voltmeter has to be connected to the two points where the voltage has to be measured

The voltmeter is connected in parallel with the circuit

Since the instrument is used to measure dc quantities, the polarity is very important

The voltmeter should have a high resistance

There is no difference between the basic instrument used to measure current and voltage since both uses a milliammeter as their basic part

The milliammeter is converted into a voltmeter by connecting it in series with a high value resistance “multiplier”

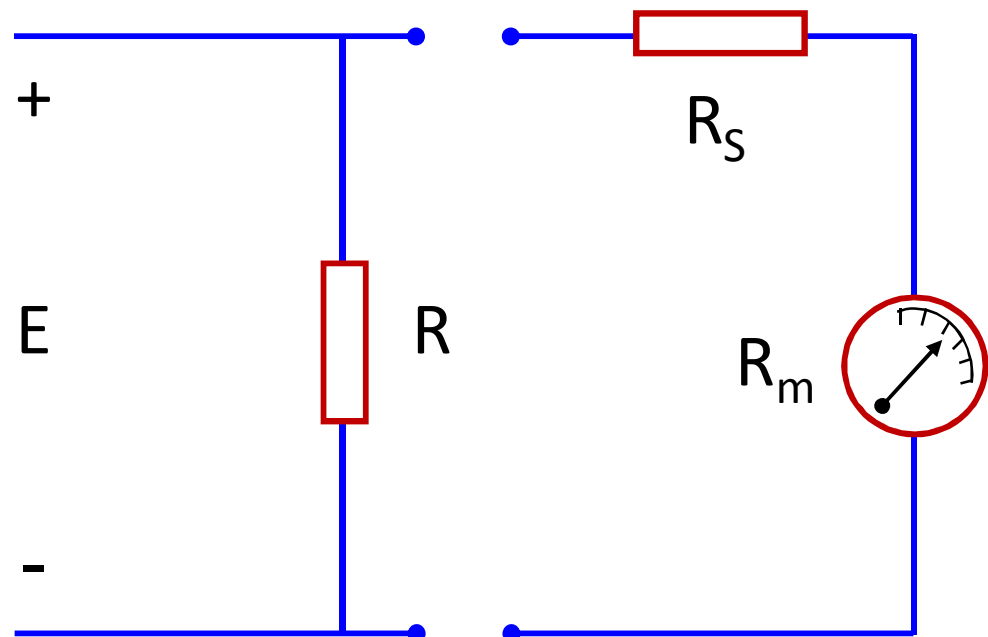
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# The moving-coil instrument as a voltmeter

With high resistance, the current flowing in the instrument will be very low

The remaining current “almost the whole current” in the original circuit will not be affected

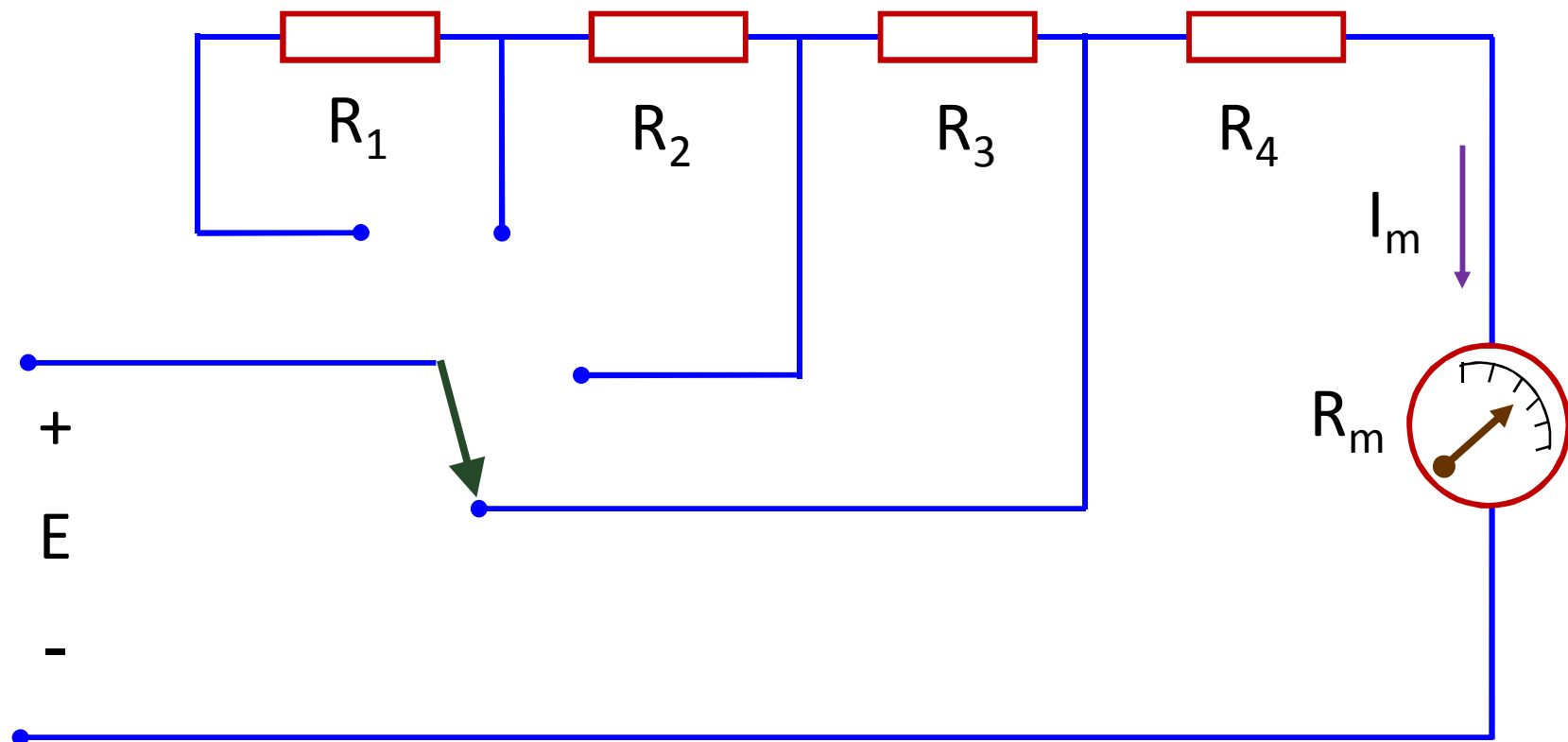
The series resistance “ $R_s$ ” has to be as high as possible to ensure the proper operation



# The moving-coil instrument as a voltmeter

To extend the operating range of the voltmeter, the multiplier has to comprise a number of resistances

Different configurations can be used to achieve this idea



# The moving-coil instrument as a voltmeter

$$E = I_m(R_S + R_m)$$

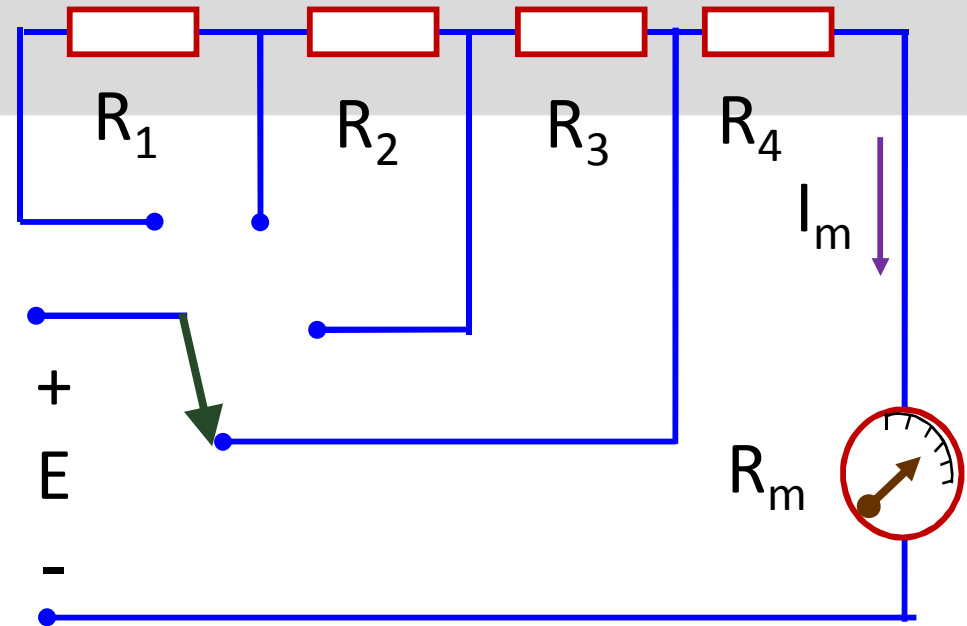
$$R_S = \frac{E}{I_m} - R_m = \frac{E - I_m R_m}{I_m}$$

Given a certain range,  
i.e.  $E = \text{range}$

$$R_S = \frac{1}{I_m} \times \text{Range} - R_m = S * \text{Range} - R_m$$

Where:  $S$  is the current sensitivity of the meter

The total voltmeter resistance =  $S * \text{range}$



# The moving-coil instrument as a voltmeter

**Example:** A PMMC instrument with full-scale deflection of  $100\ \mu\text{A}$  and a coil resistance of  $1\text{k}\Omega$  is to be converted into a voltmeter. Determine the required multiplier resistance if the voltmeter is to measure  $50\text{V}$  at full scale. Also, calculate the applied voltage when the instrument indicates 0.8, 0.5, and 0.2 of the full-scale deflection

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# The moving-coil instrument as a voltmeter

## Solution:

For  $V=50V$  full-scale deflection

$$R_s = \frac{V}{I_m} - R_m$$

$$I_m = 100\mu A$$

$$R_s = \frac{50V}{100\mu A} - 1k\Omega = 499k\Omega$$

At 0.8 full-scale deflection

$$I_m = 0.8 \times 100\mu A = 80\mu A$$

$$V = I_m (R_s + R_m) = 80\mu A (499k\Omega + 1k\Omega) = 40V$$

# The moving-coil instrument as a voltmeter

At 0.5 full-scale deflection:

$$I_m = 50\mu A$$

$$V = 50\mu A(499k\Omega + 1k\Omega) = 25V$$

At 0.2 full-scale deflection

$$I_m = 20\mu A$$

$$V = 20\mu A(499k\Omega + 1k\Omega) = 10V$$

The voltmeter total resistance =  $R_s + R_m = 500k\Omega$

Sensitivity “resistance per volt” =  $500k\Omega/50V = 10k\Omega/V$

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# The moving-coil instrument as a voltmeter

## Ohms-per-Volt Rating

Rating of analog voltmeters can be expressed in terms of the ohms of resistance required for 1 V deflection

This value is the ohms-per-volt rating “sensitivity”

It is the same for all ranges

It is determined by the full-scale current “ $I_m$ ”

$$R_V = V_{fs} * \text{ohms-per-volt rating}$$

$R_V$  : The voltmeter resistance

$V_{fs}$  : the full-scale voltage

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# The moving-coil instrument as a voltmeter

## Effect of inserting DC voltmeters

The use of voltmeter adds a parallel resistance to the circuit resulting in a reduction of the total resistance

The voltmeter current will affect the overall current in the tested circuit “voltmeter loading”

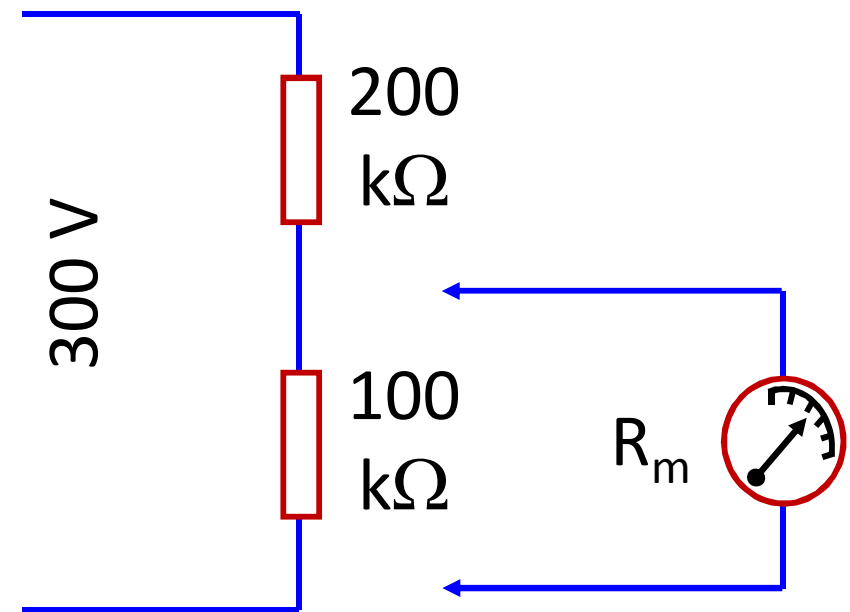
The resulting error is called a loading error

The loading effect can be reduced using a high sensitivity voltmeter “very high internal resistance”

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# The moving-coil instrument as a voltmeter

Example: A PMMC voltmeter is used to measure the voltage across the  $100\text{ k}\Omega$  resistance shown in the figure. Find the reading of the voltmeter and the percentage error if a 100 volt-range instrument is used with a sensitivity of:  $500\Omega/\text{V}$  and  $10\text{ k}\Omega/\text{V}$ .



## Solution:

Actual voltage across the 100 k $\Omega$  resistance is:

$$300 * (100 / 300) = 100 \text{ V}$$

After inserting the voltmeter:

For 500  $\Omega$ /V sensitivity:

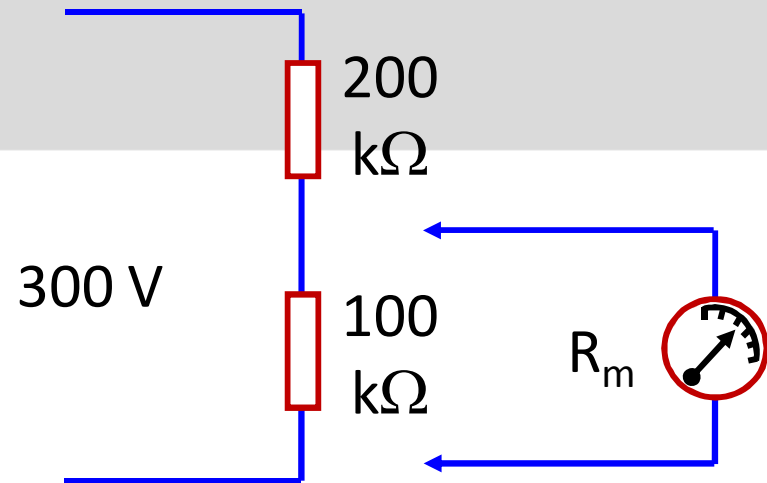
The meter resistance is:  $R_m = 500 * 100 = 50 \text{ k}\Omega$

The equivalent of the two parallel resistances is:

$$R_{eq} = 50 \text{ k}\Omega // 100 \text{ k}\Omega = 33.33 \text{ k}\Omega$$

The total resistance is:

$$R_{total} = 33.33 \text{ k}\Omega + 200 \text{ k}\Omega = 233.33 \text{ k}\Omega$$



# The moving-coil instrument as a voltmeter

## Solution:

The total current is:

$$I_{\text{total}} = 300 \text{ V} / 233.33 \text{ k}\Omega \\ = 1.285714 \text{ mA}$$

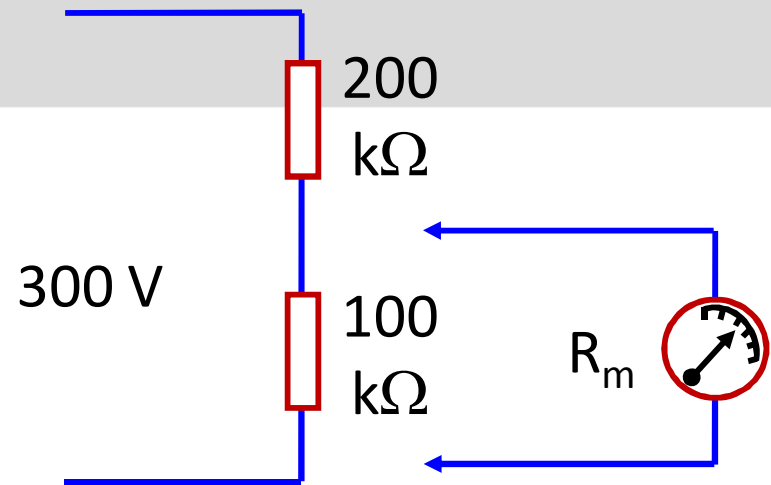
The voltage of the 200 k $\Omega$  is:

$$V_{200} = 1.285714 \text{ mA} * 200 \text{ k}\Omega = 257.1428 \text{ V}$$

The voltmeter reading is:  $V_{m1} = 300 - 257.143 = 42.86 \text{ V}$

The percentage error is given as:

$$\% \text{ error} = \frac{100 - 42.8572}{100} \times 100 = 57.1428 \%$$



# The moving-coil instrument as a voltmeter

For 10 kΩ/V sensitivity:

$$R_m = 10000 \times 100 = 1 \text{ M}\Omega$$

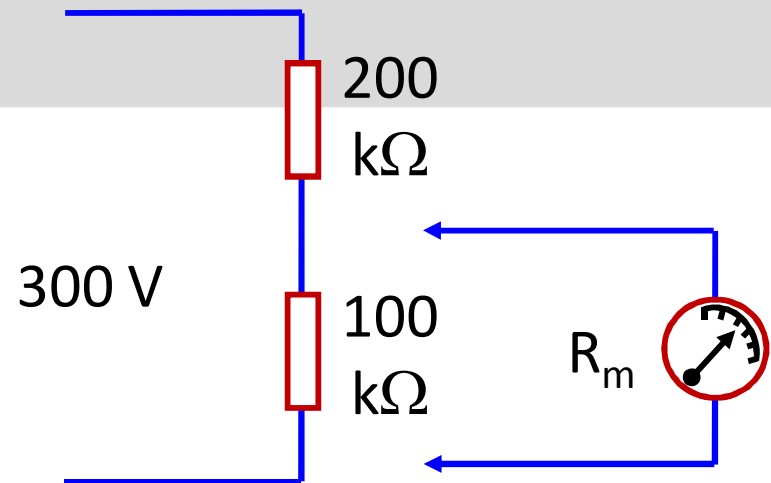
$$R_{eq} = 1 \text{ M}\Omega // 100 \text{ k}\Omega \\ = 90.91 \text{ k}\Omega$$

$$R_{total} = 90.91 \text{ k}\Omega + 200 \text{ k}\Omega \\ = 290.91 \text{ k}\Omega$$

$$I_{total} = 300 \text{ V} / 290.91 \text{ k}\Omega = 1.03125 \text{ mA}$$

$$V_{200} = 1.03125 \text{ mA} \times 200 \text{ k}\Omega = 206.25 \text{ V}$$

$$V_{m1} = 300 - 206.25 = 93.75 \text{ V}$$



$$\% \text{ error} = \frac{100 - 93.75}{100} \times 100 = 6.25 \%$$